

Chiral Topological Superconductivity, Charge-4e Superconductivity, and Chiral Metal in the Twisted-Bilayer Quasicrystals

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Outline

- Introduction
- Twisted-bilayer quasi-crystals carrying chiral TSC
- Low-energy effective model for phase fluctuations
- The charge-4e SC & chiral metal
- Discussions & Conclusion

Ref: Y.B.Liu, et al, arXiv: 2301.06357;
Y.B.Liu, et al, arXiv: 2301.07553;
Y.B.Liu, et al, Phys. Rev. B 107, 014501 (2023)

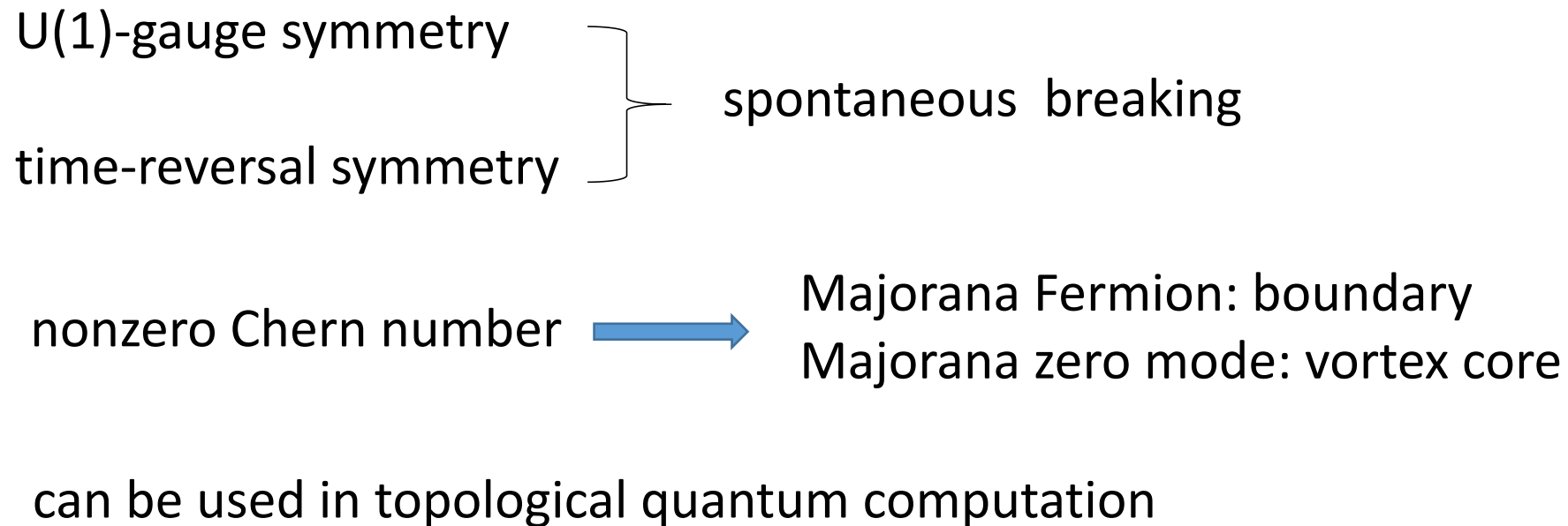
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Introduction

- Chiral topological superconductivity (TSC)

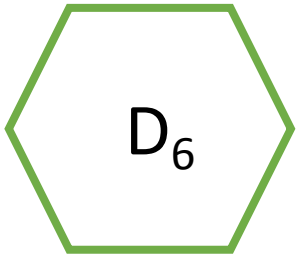


Introduction

- Mechanism: the 1:i mixing of two degenerate pairings (2D IRRP of a point-group)



E_1 IRRP : $(p_x, p_y) \longrightarrow p_x + ip_y, p_x - ip_y$ (p+ip)



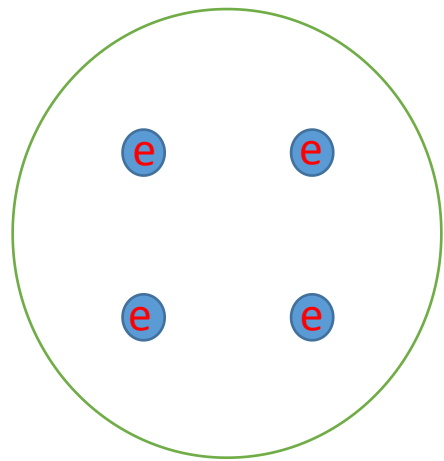
E_1 IRRP : $(p_x, p_y) \longrightarrow (p + ip)$; E_2 IRRP : $(d_x^2 - y^2, d_{xy}) \longrightarrow (d + id)$

- The material realization of chiral TSC is still a big challenge

candidates: Sr_2RuO_4 ; 1/4-doped graphene; MA-TBG (to be verified)

Introduction

- The charge-4e SC



4e -- bound state



BEC
condensation




SC with
fractionalized flux

half flux quanta:
 $hc/4e$

- Can be realized as vestigial phase of degenerate-pairing ground state

Introduction

- Incommensurate PDW fluctuation  charge-4e SC & CDW
(U(1)*U(1) symmetry breaking)

$$\Delta(\mathbf{r}) = \Delta_{\mathbf{Q}}(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} + \Delta_{-\mathbf{Q}}(\mathbf{r}) e^{-i\mathbf{Q}\cdot\mathbf{r}}$$

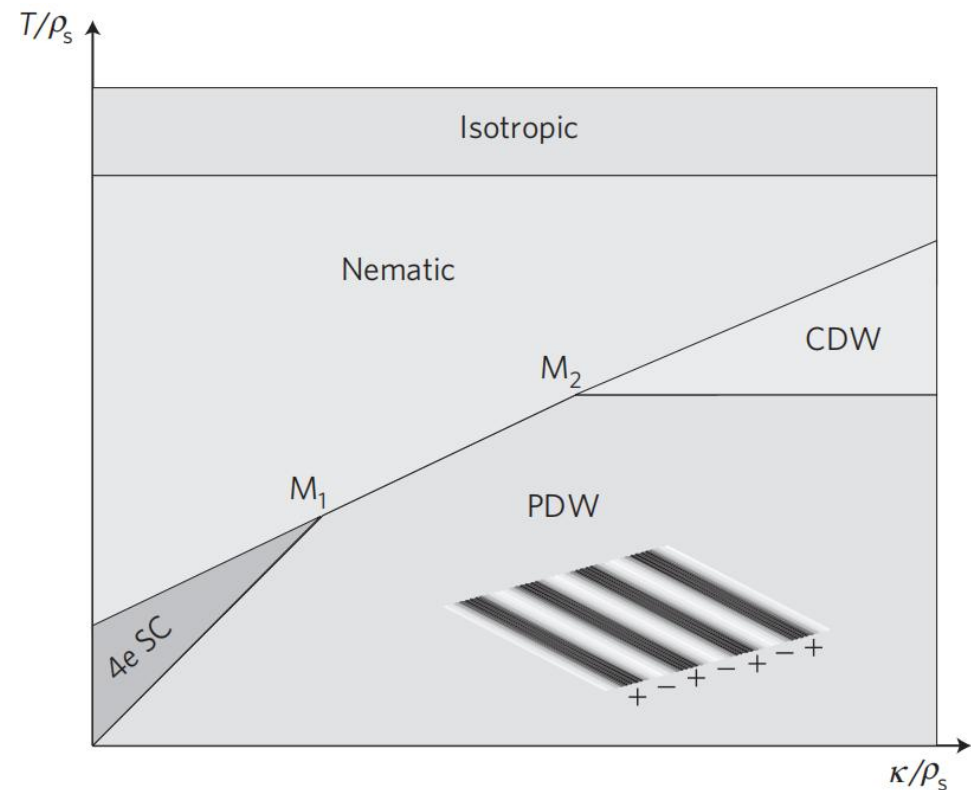
$$\Delta_{\pm\mathbf{Q}}(\mathbf{r}) = \Delta_{\text{SC}} \exp\{i(\theta(\mathbf{r}) \pm \phi(\mathbf{r}))\}$$

$$\Delta_{4e}(\mathbf{r}) \propto \Delta_{-\mathbf{Q}}(\mathbf{r}) \Delta_{\mathbf{Q}}(\mathbf{r}) = \Delta_{4e} \exp[i2\theta(\mathbf{r})]$$

$$\rho_{\mathbf{K}}(\mathbf{r}) \propto \Delta_{-\mathbf{Q}}^*(\mathbf{r}) \Delta_{\mathbf{Q}}(\mathbf{r}) = \rho_{\mathbf{K}} \exp[i2\phi(\mathbf{r})]$$

$\theta(\mathbf{r})$ -- unilateral ordering: charge-4e SC

$\phi(\mathbf{r})$ -- unilateral ordering: CDW with $\mathbf{K} = 2\mathbf{Q}$



E. Berg, et al, Nat. Phys. 5, 830 (2009).

Introduction

- Nematic pairing fluctuation \longrightarrow charge-4e SC & nematic metal (U(1)*Z(3) SB)

$$\Delta = (\Delta_x, \Delta_y)^T : E_u \text{ IRRP of } D_{3d}$$

$$\Delta_{\pm} = \Delta_x \pm i\Delta_y : \text{basis trsf}$$

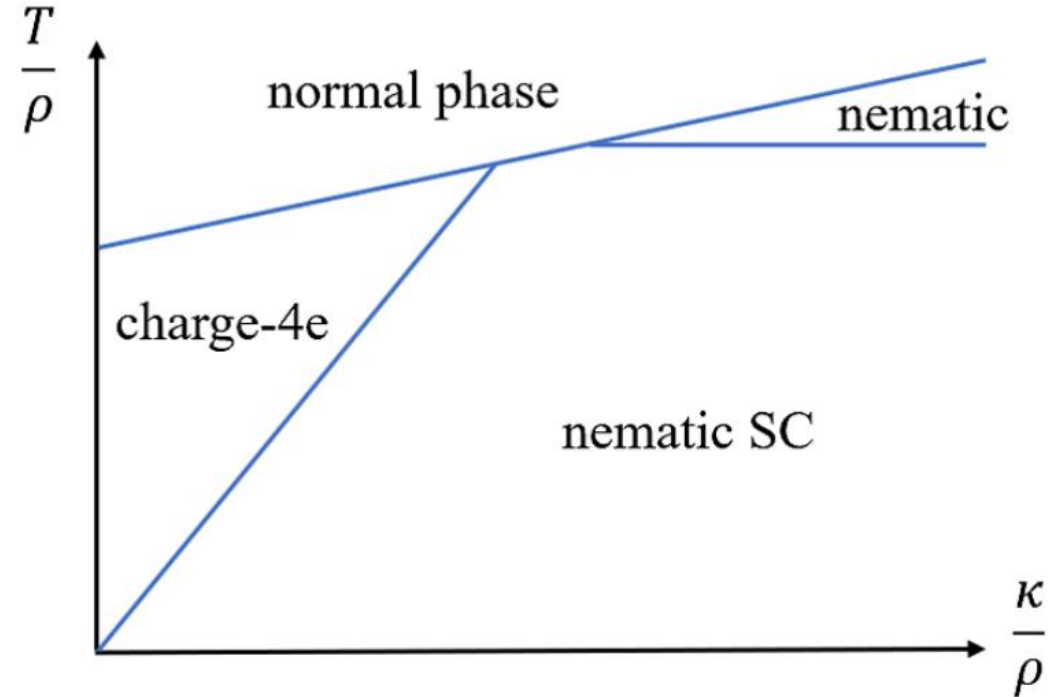
$$\Delta_+ = |\Delta|e^{i(\theta+\phi)}, \quad \Delta_- = |\Delta|e^{i(\theta-\phi)}$$

$$\Delta_{4e} \sim \Delta_+ \Delta_- \rightarrow \Delta_{4e} e^{2i\theta}$$

$$Q \sim \Delta_-^\dagger \Delta_+ \rightarrow Q e^{2i\phi}$$

$\theta(\mathbf{r})$ -- unilateral ordering: charge-4e SC





$\phi(\mathbf{r})$ -- unilateral ordering: nematic metal



S.K.Jian, et al, PRL127,22701 (2021).

R. M. Fernandes, et al, PRL127,047001 (2021)

Main Idea

- Charge-4e SC as a vestigial phase of multi-component pairing state.
- Extra symmetry breaking besides U(1)-gauge in the ground state
PDW: translation ; nematic SC: rotation ;
unilateral U(1)-gauge breaking  charge-4e SC
unilateral extra symmetry breaking: CDW, nematic order
- The TB-QC consists of two superconducting monolayers
interlayer Josephson coupling  chiral TSC
- Vestial phases of the chiral TSC ($U(1)*Z_2$ SB)
total phase order  charge-4e SC ($U(1)$ -gauge SB)
relative phase order  chiral metal (Z_2 time-reversal SB)

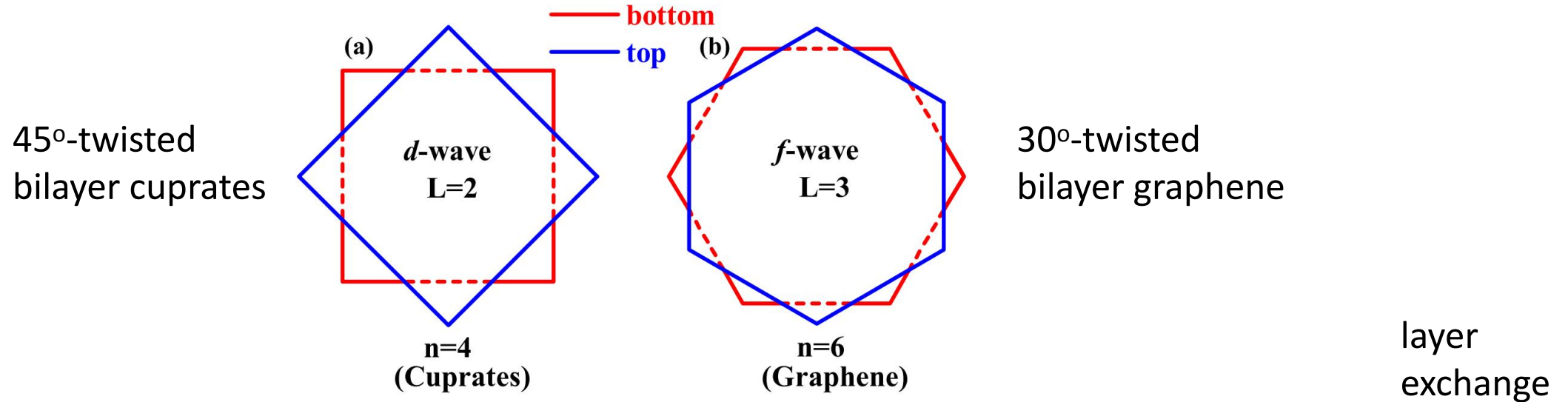
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Twisted-Bilayer Quasi-Crystal (TB-QC)

- Two D_n -symmetric monolayers stacked with the largest twist angle π / n



- Moireless quasi-crystal structure.
- Enlarged point group $D_{nd} \sim D_{2n}$. New symmetry generator $\tilde{C}_{2n}^1 \equiv C_{2n}^1 * P$
- Suppose : each monolayer carries SC with angular momentum $L = n / 2$
- What pairing can be driven by interlayer Josephson Coupling (IJC) in the TB-QC?

Ginzburg-Landau Theory

- Pairing gap $\Delta^{(\mu)}(\mathbf{k})$; fixed form factor $\Gamma^{(\mu)}(\mathbf{k})$; complex amplitude ψ_μ

$$\Delta^{(\mu)}(\mathbf{k}) = \psi_\mu \Gamma^{(\mu)}(\mathbf{k}), \quad \mu = \text{t/b (top/bottom)}$$

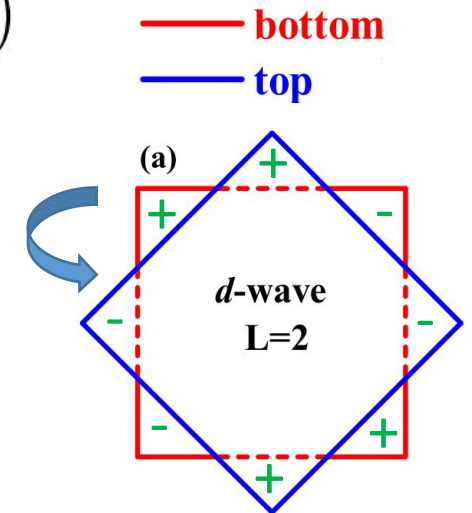
- The fixed form factor $\Gamma^{(\mu)}(\mathbf{k})$ for $L = n/2$ satisfies

$$\Gamma^{(\text{b})}(\mathbf{k}) = \hat{P}_{\frac{\pi}{n}} \Gamma^{(\text{t})}(\mathbf{k}), \quad \hat{P}_{\frac{2\pi}{n}} \Gamma^{(\mu)}(\mathbf{k}) = -\Gamma^{(\mu)}(\mathbf{k})$$

- The G-L free energy functional

$$F(\psi_t, \psi_b) = F_0(|\psi_t|^2) + F_0(|\psi_b|^2) + F_J(\psi_t, \psi_b)$$

$$F_J(\psi_t, \psi_b) = F_J^{(1)}(\psi_t, \psi_b) + F_J^{(2)}(\psi_t, \psi_b) + \dots$$



Ginzburg-Landau Theory

- The first-order IJC : satisfies U(1)- and TR

$$F_J^{(1)}(\psi_t, \psi_b) = -A(\psi_t \psi_b^* + c.c)$$

- Invariance of $F_J^{(1)}$ under \tilde{C}_{2n}^1 dictates $A=0$

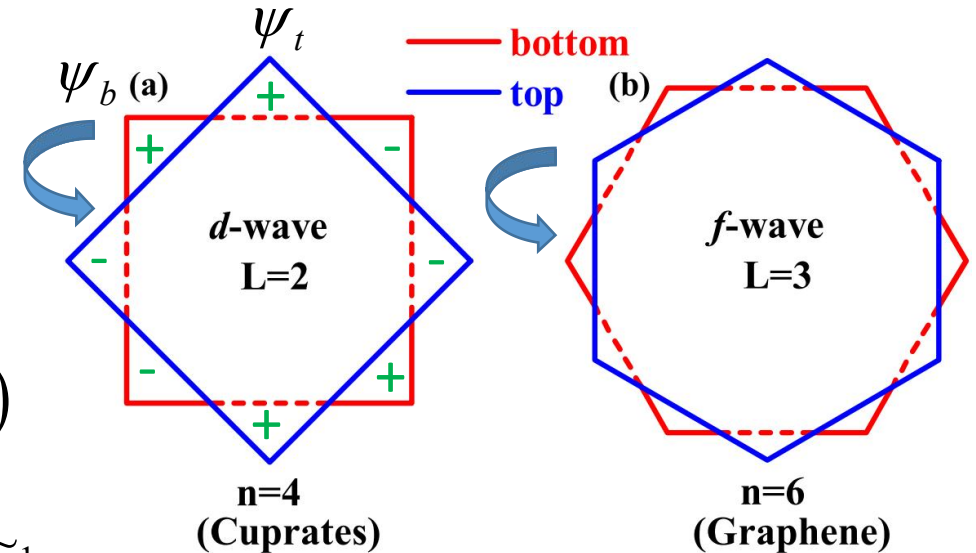
$$\tilde{C}_{2n}^1 : \left. \begin{array}{l} \psi_b \rightarrow \psi_t \\ \psi_t \rightarrow -\psi_b \end{array} \right\} \Rightarrow F_J^{(1)}(\psi_t, \psi_b) \rightarrow -F_J^{(1)}(\psi_t, \psi_b)$$

- The second-order IJC: satisfies U(1), TR, \tilde{C}_{2n}^1

$$F_J(\psi_t, \psi_b) = A_0(\psi_t^2 \psi_b^{2*} + c.c.)$$

- Minimization of free energy:

$\psi_t = \pm i \psi_b,$	$A_0 > 0,$	\longrightarrow	chiral	d+id/f+if	TSC	} determined by microscopic calc
$\psi_t = \pm \psi_b,$	$A_0 < 0.$	\longrightarrow	nematic	d/f	SC	

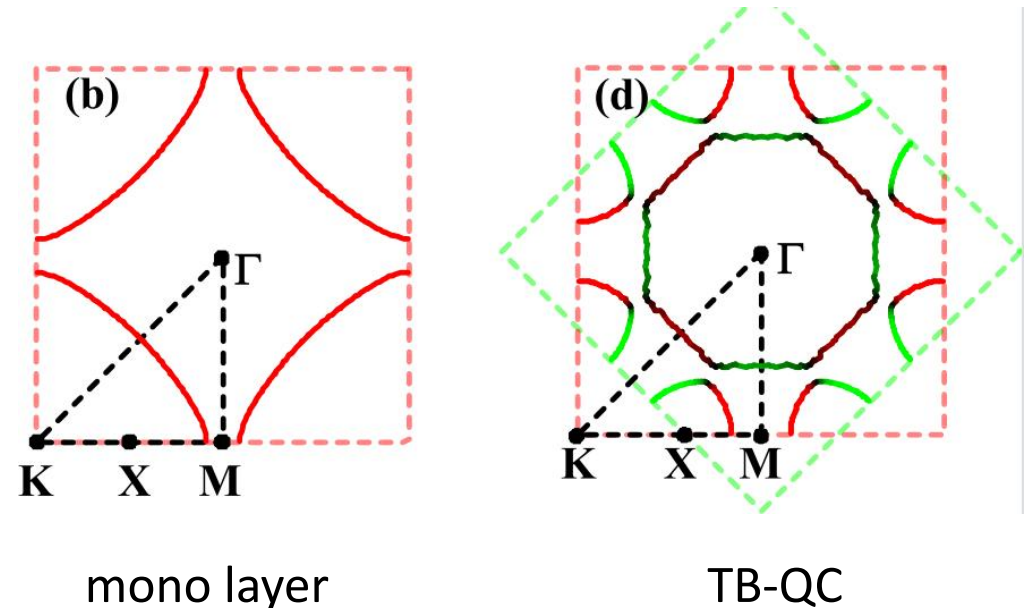
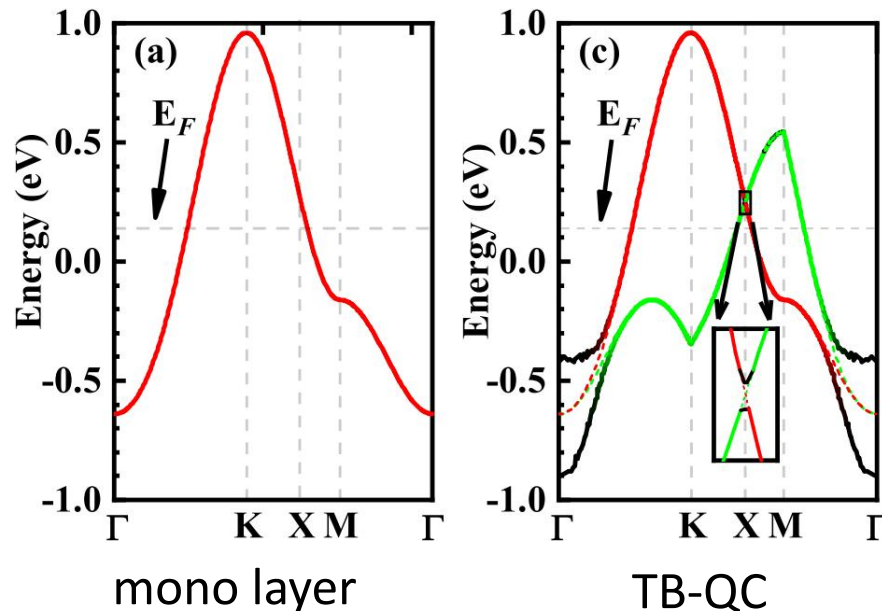


Microscopic Calculations

- The Moireless QC structure disables the standard band-theory study.
- Weak interlayer coupling \longrightarrow perturbational-band theory

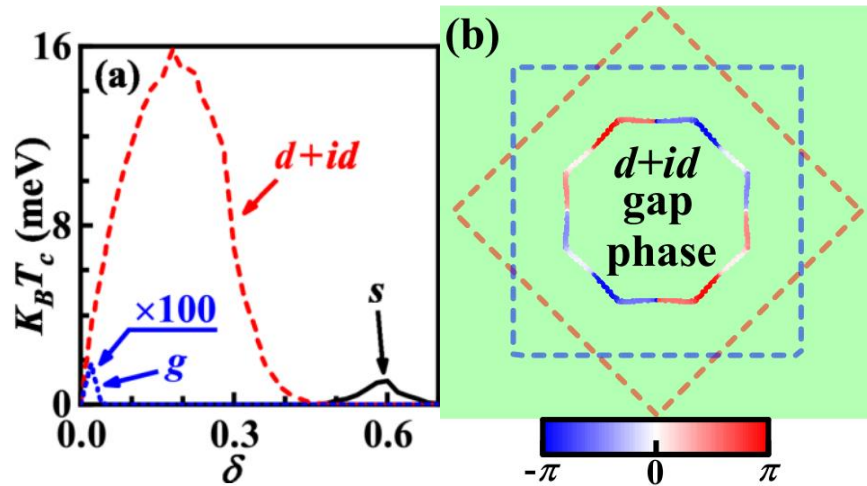
$$H_0 = \sum_{\mathbf{k}\mu\alpha\sigma} c_{\mathbf{k}\mu\alpha\sigma}^\dagger c_{\mathbf{k}\mu\alpha\sigma} \varepsilon_{\mathbf{k}}^{\mu\alpha}, \quad H' = \sum_{\mathbf{k}\mathbf{q}\alpha\beta\sigma} c_{\mathbf{k}\mathbf{t}\alpha\sigma}^\dagger c_{\mathbf{q}\mathbf{b}\beta\sigma} T_{\mathbf{k}\mathbf{q}}^{\alpha\beta} + h.c.$$

- The interlayer H' -term as perturbation: band structures and FSs

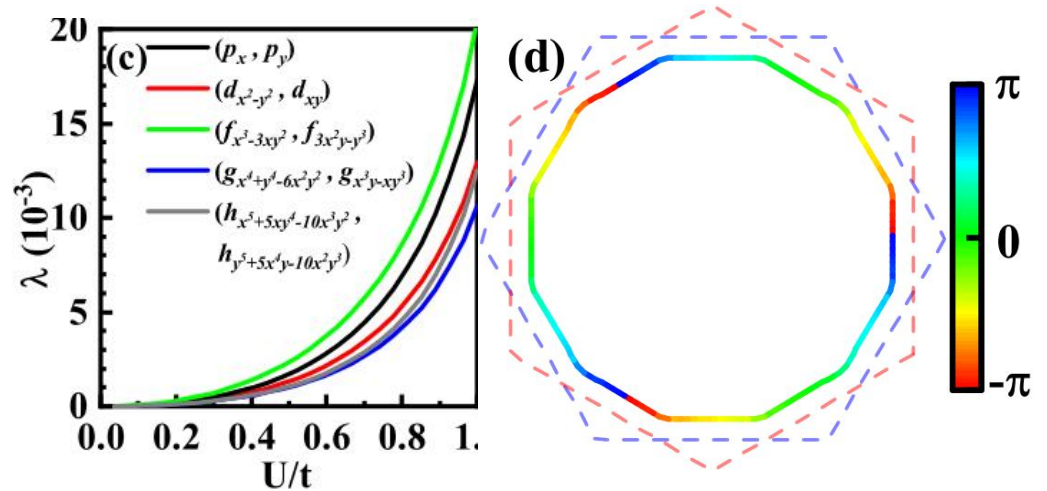


Microscopic Calculations

- The t-J or Hubbard interaction added: Gutzwiller MF or RPA treatment
- Effective interactions projected on the perturbation-corrected FS
- Solved gap equation classified by irreducible representation of D_8/D_{12}
- The doubly degenerate pairing: always 1:i mixing to minimize energy



$d+id$ TSC in 45-twisted bilayer cuprates



$f+if$ TSC in 30-twisted bilayer graphene ($\delta = 0.15$)

- The chiral $d+id$ / $f+if$ TSC realized from d / f -wave SC in each monolayer.

Y.B.Liu, et al, arXiv: 2201.01656; ibid 2301.07553; ibid, Phys. Rev. B 107, 014501 (2023)

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Y.B.Liu, et al, Phys. Rev. B 107, 014501 (2023)

Low-Energy Effective Model

- Fix $\Gamma^{(\mu)}$, set $\psi_\mu \rightarrow \psi_\mu(\mathbf{r})$: spatial fluctuation driven by temperature
- Focus on the phase fluctuation $\psi_{t/b} = \psi_0 e^{i\theta_{t/b}(\mathbf{r})}$ $\psi_0 > 0$
- The total- and relative- phase fields

$$\theta_t(\mathbf{r}) = \theta_+(\mathbf{r}) + \theta_-(\mathbf{r}) , \quad \theta_b(\mathbf{r}) = \theta_+(\mathbf{r}) - \theta_-(\mathbf{r})$$

- The low-energy effective Hamiltonian

$$H = H_0 [\partial_\pm \theta_+, \partial_\pm \theta_-] + A_0 \psi_0^4 \int \cos 4\theta_-(\mathbf{r}) d^2\mathbf{r} \quad (\partial_\pm \equiv \partial_x \pm i\partial_y)$$

$$H_0 = \frac{1}{2} \int d^2\mathbf{k} [\theta_+(\mathbf{k}) \theta_+(-\mathbf{k}) (\alpha k_+^2 + \beta k_-^2 + \rho k_+ k_-) \\ + \theta_+(\mathbf{k}) \theta_-(-\mathbf{k}) (\gamma k_+^2 + \delta k_-^2 + \eta k_+ k_-) \\ + \theta_-(\mathbf{k}) \theta_-(-\mathbf{k}) (\epsilon k_+^2 + \xi k_-^2 + \kappa k_+ k_-)] .$$

Low-Energy Effective Model

- Symmetry constraint: Under \tilde{C}_{2n}^1 , $\psi_{t/b}(\mathbf{r}) \rightarrow \tilde{\psi}_{t/b}(\mathbf{r})$

$$\tilde{\psi}_b(\mathbf{r}) = \psi_t\left(\hat{P}_{\frac{\pi}{n}}^{-1}\mathbf{r}\right), \quad \tilde{\psi}_t(\mathbf{r}) = -\psi_b\left(\hat{P}_{\frac{\pi}{n}}^{-1}\mathbf{r}\right)$$

$$\theta_+(\mathbf{k}) \rightarrow \theta_+\left(\hat{P}_{\frac{\pi}{n}}^{-1}\mathbf{k}\right), \quad \theta_-(\mathbf{k}) \rightarrow -\theta_-\left(\hat{P}_{\frac{\pi}{n}}^{-1}\mathbf{k}\right)$$

- Invariance of Hamiltonian H under \tilde{C}_{2n}^1 , requires

$$\alpha = \beta = \gamma = \delta = \eta = \varepsilon = \xi = 0$$

- The resultant Hamiltonian

$$H = \int d^2\mathbf{r} \left(\frac{\rho}{2} |\nabla\theta_+|^2 + \frac{\kappa}{2} |\nabla\theta_-|^2 + A_0\psi_0^4 \cos 4\theta_- \right)$$

ρ, κ : stiffness parameters

Low-Energy Effective Model

- Three types of vortices

$$\theta_t(\mathbf{r}) = \theta_+(\mathbf{r}) + \theta_-(\mathbf{r}), \quad \theta_b(\mathbf{r}) = \theta_+(\mathbf{r}) - \theta_-(\mathbf{r})$$

$$\oint \nabla \theta_{t/b}(r) \cdot d\vec{l} = 2\pi n_{t/b}, n_{t/b} \in I; \quad \oint \nabla \theta_{+/-}(r) \cdot d\vec{l} = 2\pi n_{+/-}, n_{+/-} = (n_t \pm n_b)/2$$

$$n_{\pm} \in I \neq 0 \Rightarrow \text{integer vortex}, \quad n_{\pm} \in I + 1/2 \Rightarrow \text{half-half vortex}$$

- Various phases and phase diagram

Y.B.Liu, et al, arXiv: 2301.06357

For $T \rightarrow 0$, all types of vortex--anti-vortex pairs \longrightarrow chiral TSC

When $T \uparrow$,
 θ_- vortices first unilateral proliferate: restore TRS \longrightarrow charge-4e SC
 θ_+ vortices first unilateral proliferate: kill SC \longrightarrow chiral metal
 half θ_+ - half θ_- vortices first proliferate: kill SC and restore TRS \longrightarrow metal

Which phase is realized: determined by T and $\kappa/\rho \longrightarrow$ phase diagram

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Y.B.Liu, et al, Phys. Rev. B 107, 014501 (2023)

The RG Study

- The action describing the pairing-phase fluctuations

$$S = \int d^2\mathbf{r} \left(\frac{\rho}{2T} |\nabla\theta_+|^2 + \frac{\kappa}{2T} |\nabla\theta_-|^2 + g_4 \cos 4\theta_- \right)$$

- Map to the Sine-Gordon model via the dual representation

$$S_{\text{SG}} = \int d^2\mathbf{x} \left(\frac{T}{2\rho} |\nabla\tilde{\theta}_+|^2 + \frac{T}{2\kappa} |\nabla\tilde{\theta}_-|^2 + g_4 \cos 4\theta_- - g_{2,0} \right. \\ \left. \times \cos 2\pi\tilde{\theta}_+ - g_{0,2} \cos 2\pi\tilde{\theta}_- - g_{1,1} \cos \pi\tilde{\theta}_+ \cos \pi\tilde{\theta}_- \right)$$

$\tilde{\theta}_+, \tilde{\theta}_-$: vortex fields of θ_+, θ_-

$g_{2,0}, g_{0,2}, g_{1,1}$: fugacities of θ_+ vortices, θ_- vortices, combined half θ_+ - half θ_- vortices

The one-loop RG Study

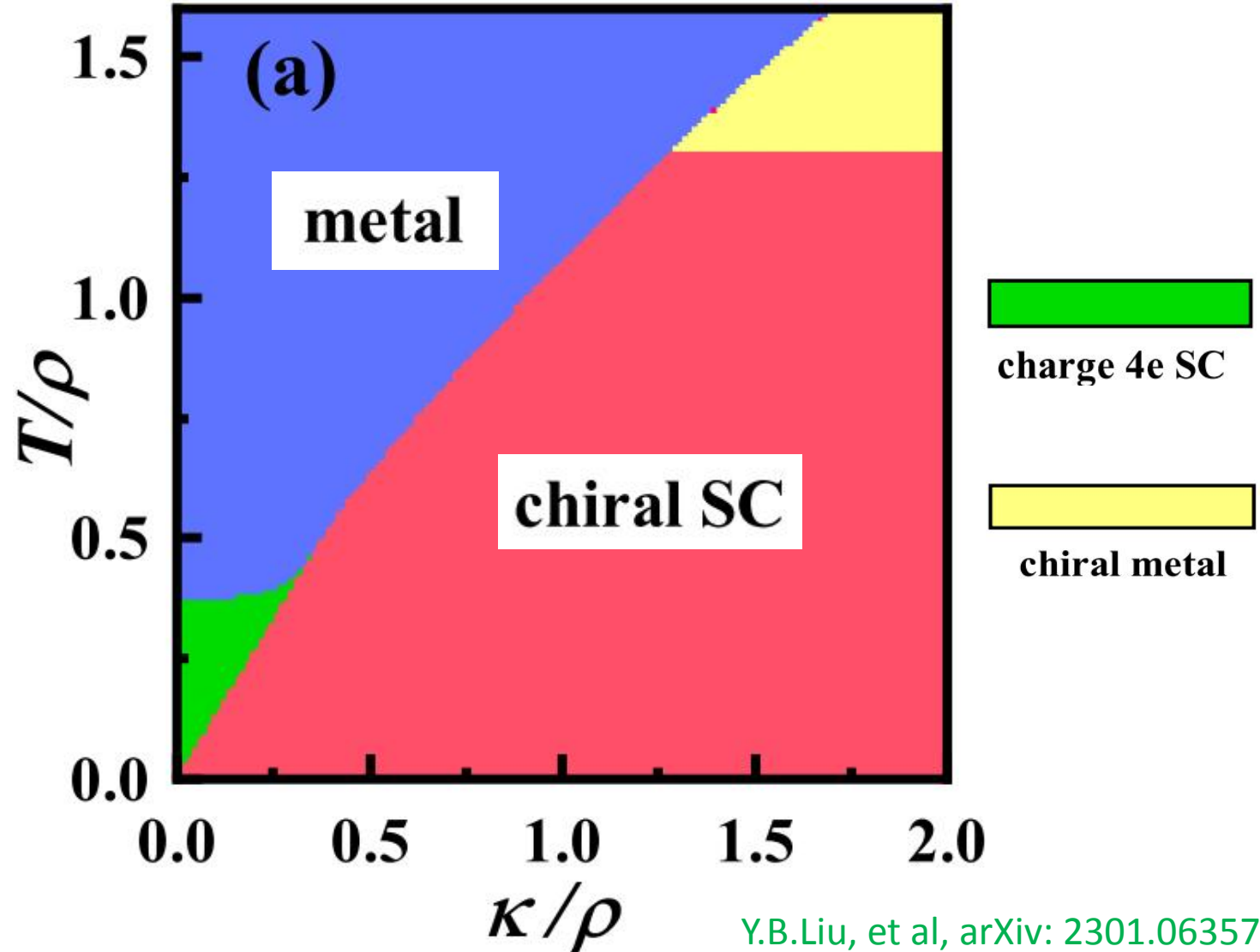
$$\frac{dg_{2,0}}{d \ln b} = (2 - \pi \rho') g_{2,0}$$

$$\frac{dg_{0,2}}{d \ln b} = (2 - \pi \kappa') g_{0,2}$$

$$\frac{dg_{1,1}}{d \ln b} = \left(2 - \frac{\pi}{4}(\rho' + \kappa')\right) g_{1,1}$$

$$\frac{dg_4}{d \ln b} = \left(2 - \frac{4}{\pi \kappa'}\right) g_4$$

$g_{2,0}$	$g_{0,2}$	g_4	$g_{1,1}$	phase
∞	∞	0	∞	normal
∞	0	0	∞	normal
0	0	0	∞	normal
0	∞	0	∞	normal
∞	∞	0	0	normal
0	∞	0	0	charge 4e SC
0	0	∞	0	chiral SC
∞	0	∞	0	chiral metal



The Monte-Carlo Study

- Calculate physical quantities by MC for the discretized Hamiltonian
- The superfluid density S characterizing SC and the Ising ODP I for TRSB
- The susceptibility χ_{\pm} and correlation function $\eta_{\pm}(\Delta\mathbf{r})$ for the total- and relative- pairing phases, related to SC and TRSB.

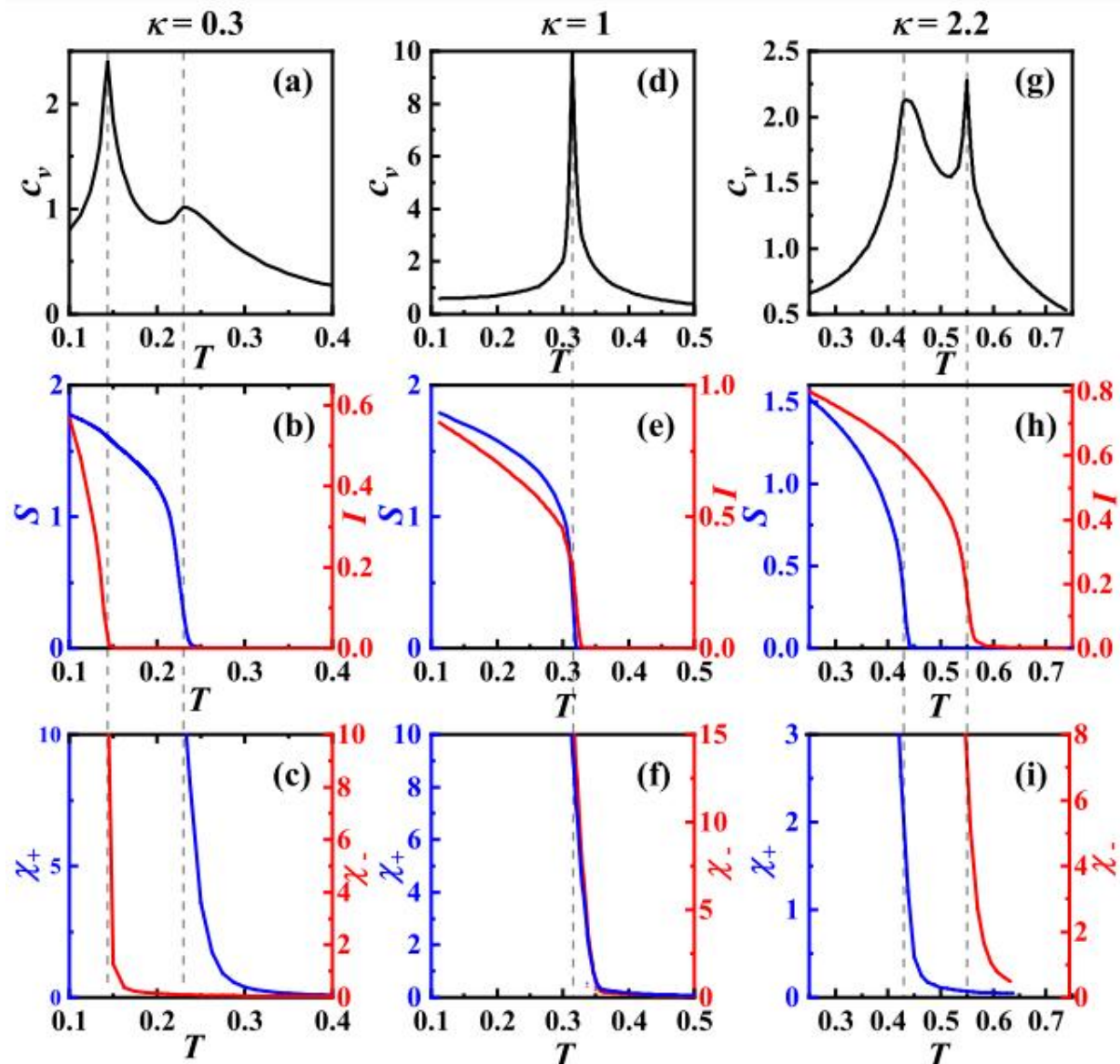
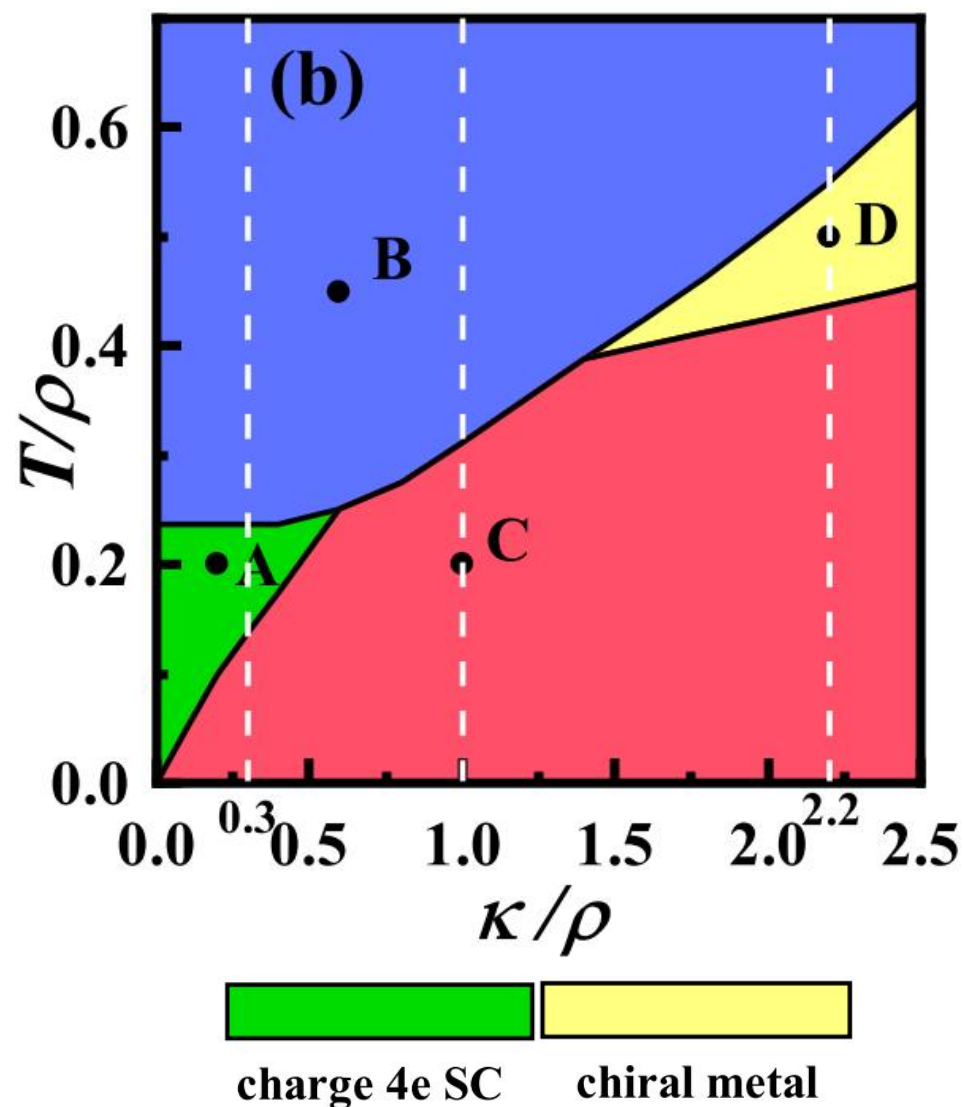
$$S = \frac{1}{N} \frac{\partial^2 F(\Phi)}{\partial \Phi^2} \quad , \quad \Phi: \text{the insert flux}; \quad F: \text{the free energy}$$

$$I \equiv \frac{1}{N^2} \sum_{i,j} \langle \sin[\theta_t(\mathbf{r}_i) - \theta_b(\mathbf{r}_i)] \cdot \sin[\theta_t(\mathbf{r}_j) - \theta_b(\mathbf{r}_j)] \rangle$$

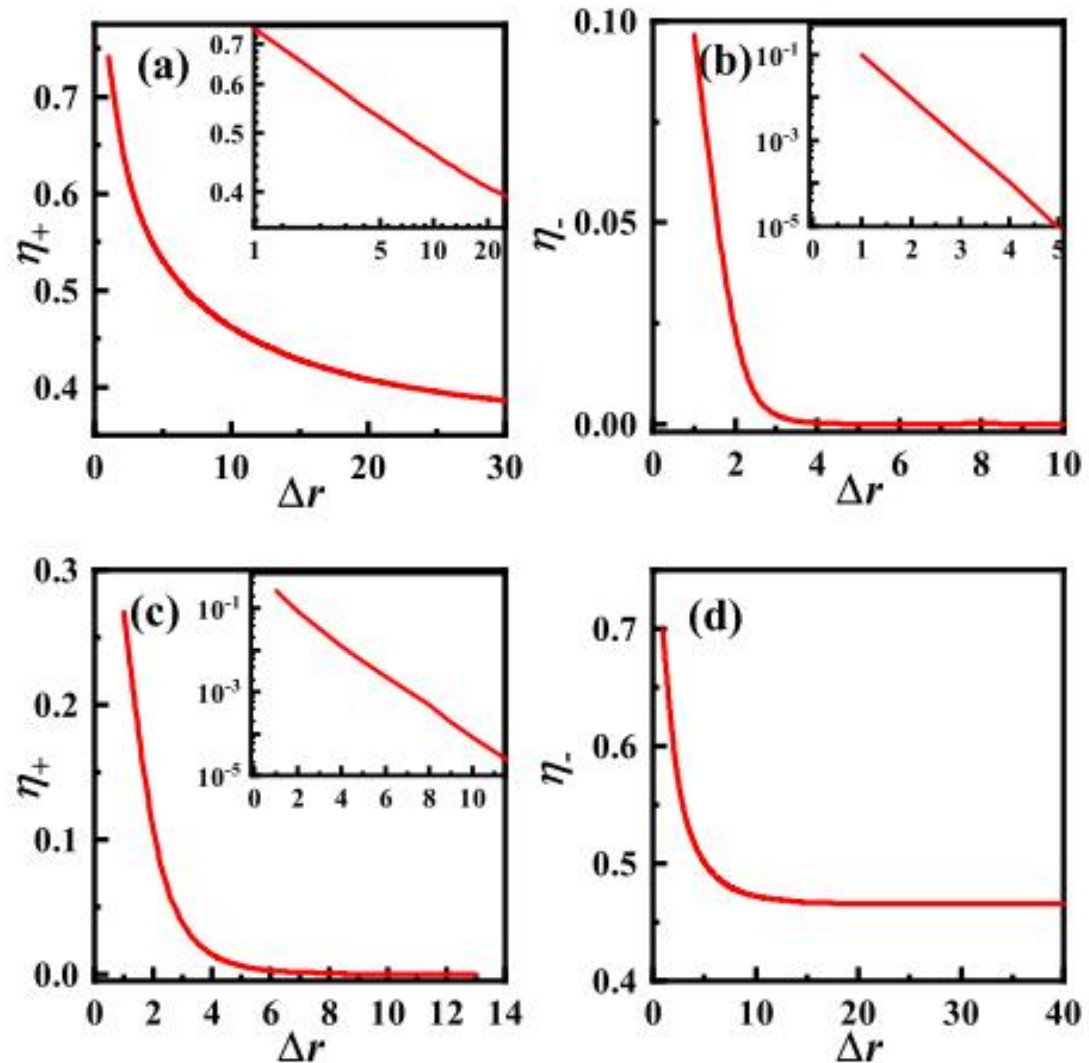
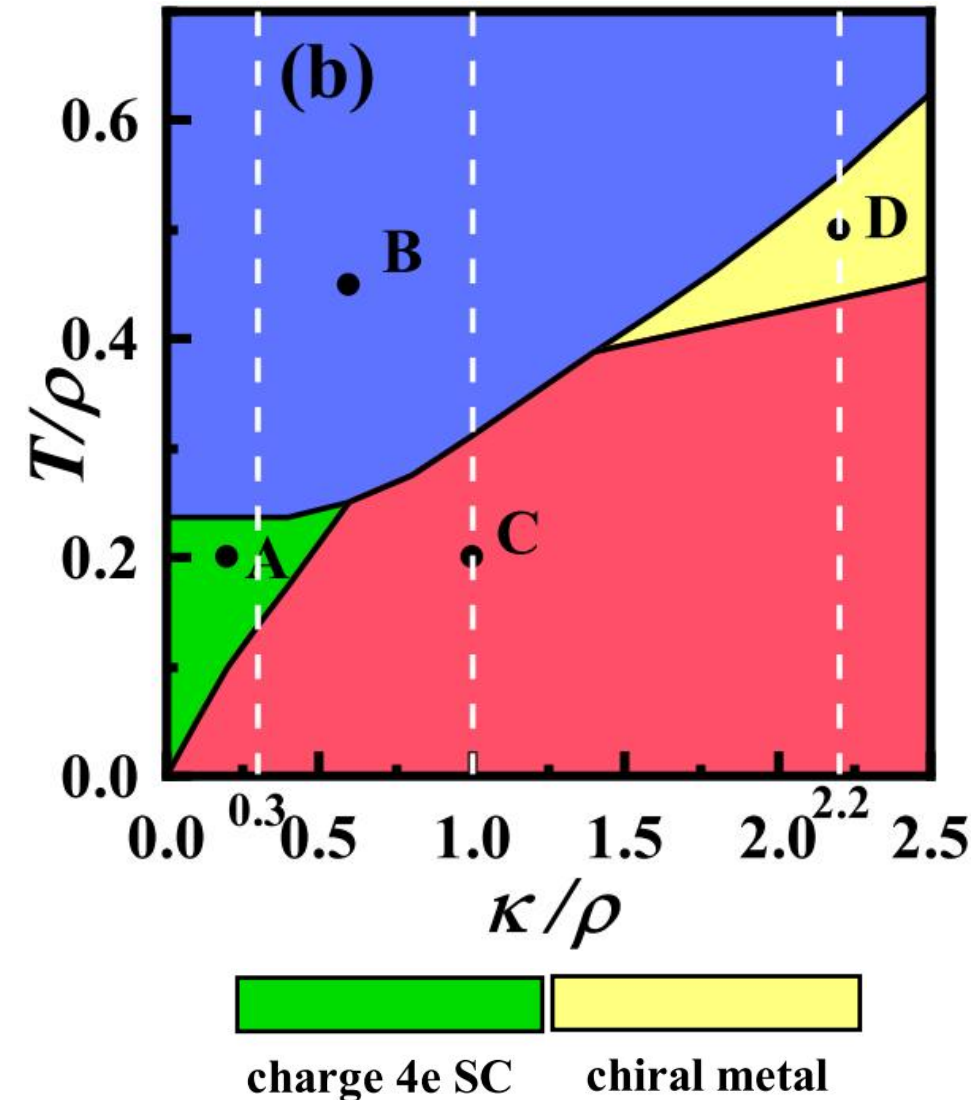
$$\chi_{\pm} \equiv \frac{1}{NT} \sum_i \left\langle \left| e^{i[\theta_t(\mathbf{r}_i) \pm \theta_b(\mathbf{r}_i)]} \right|^2 \right\rangle$$

$$\eta_{\pm}(\Delta\mathbf{r}) = \frac{1}{N} \sum_{\mathbf{r}} \left\langle e^{i[\theta_t(\mathbf{r}) \pm \theta_b(\mathbf{r}) - \theta_t(\mathbf{r} + \Delta\mathbf{r}) \mp \theta_b(\mathbf{r} + \Delta\mathbf{r})]} \right\rangle$$

The Monte-Carlo Study



The Monte-Carlo Study



point A

point D

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Y.B.Liu, et al, Phys. Rev. B 107, 014501 (2023)

Comparison with charge-4e SC in heterostructure bilayer

- Charge-4e SC in one layer induced by charge-2e SC in the other through IJC.

$$H = -J_1 \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j) - J_2 \sum_{\langle i,j \rangle} \cos(\varphi_i - \varphi_j) + K \sum_i \cos(2\theta_i - 2\varphi_i),$$

$J_{1/2}$: phase stiffness in layer 1/2; $J_2 < J_1$

K : second-order IJC

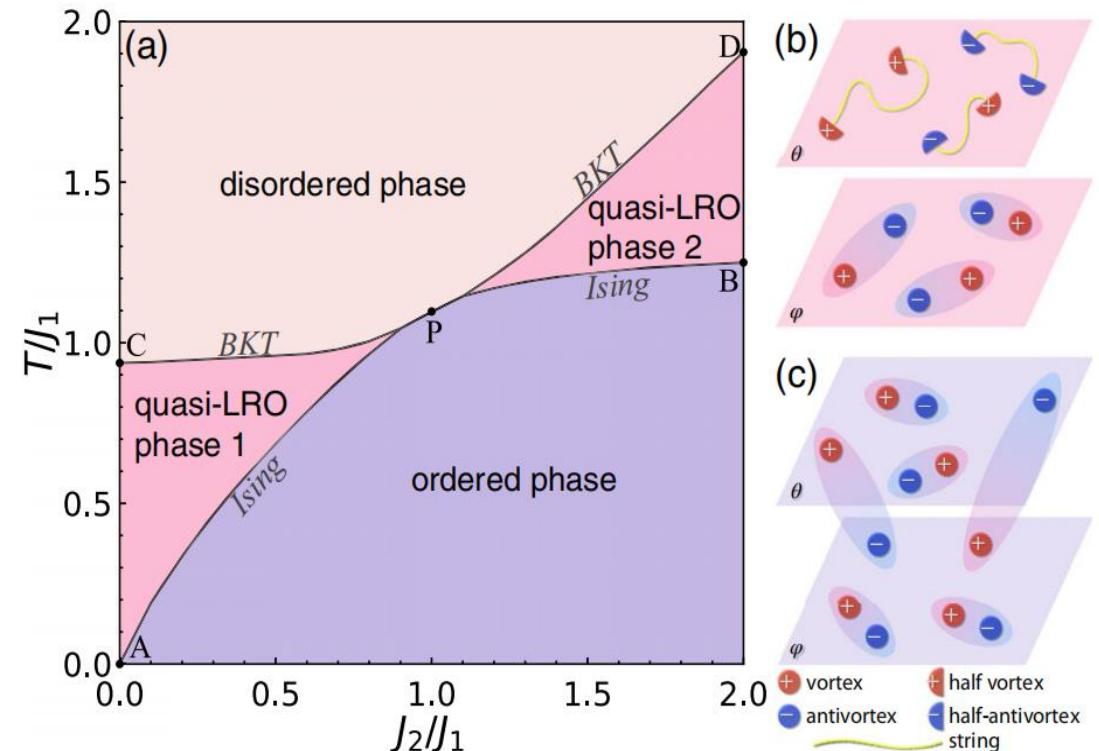
For $T \rightarrow 0$, both layers host SC

When $T \uparrow$, SC2 is killed, SC1 maintains

$\theta(\mathbf{r})$ -- unilateral quasi-ordering: SC1

$\phi(\mathbf{r})$ -- Z_2 fluctuation by the IJC: kill SC2

$\phi(\mathbf{r}) * 2$ -- quasi-ordering: charge-4e SC in layer2



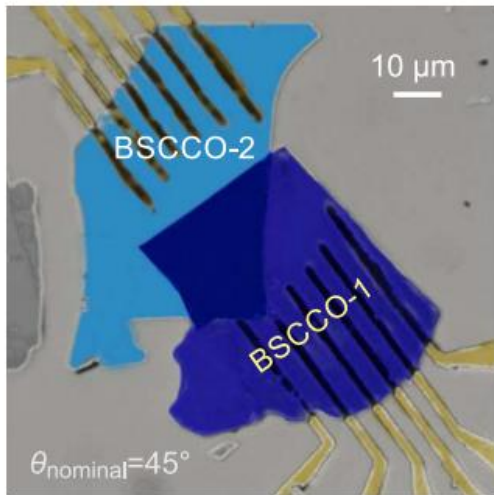
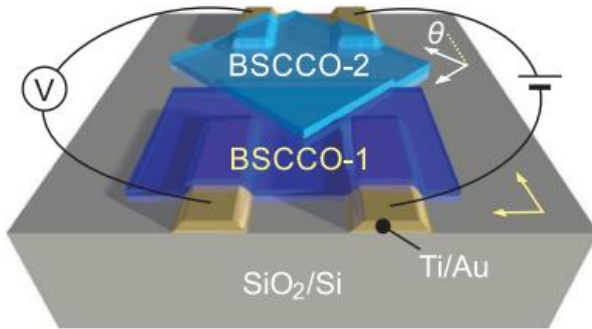
F. F. Song, et al, PRL128, 195301 (2022)

Comparison with Other chiral TSCs

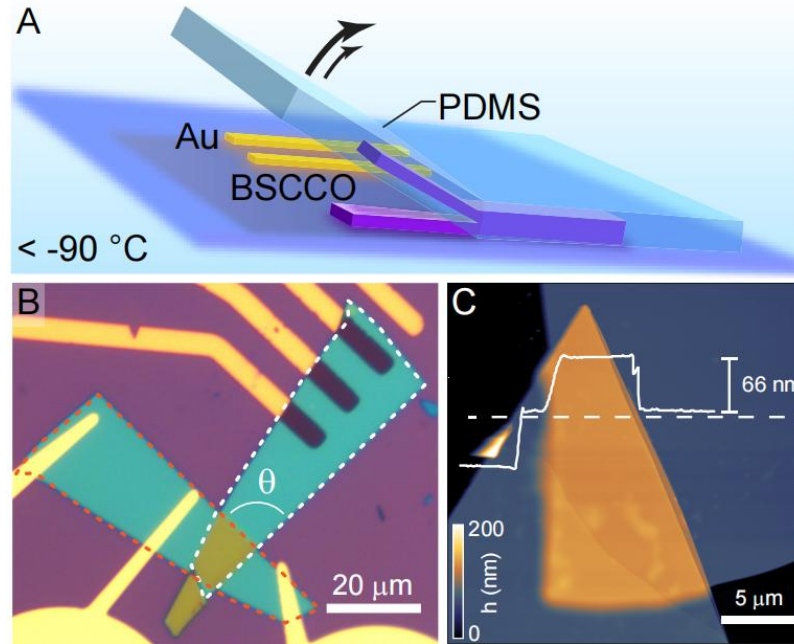
- The p+ip/d+id chiral TSCs can emerge on conventional D_4 or D_6 lattices.
- U(1)-gauge and TRSB might also be unilateral broken to get vestigial phases
- The symmetry allows such term to appear in the Hamiltonian density
$$\nabla_{\pm}\theta_{+} \cdot \nabla_{\pm}\theta_{-}$$
- Such dynamic coupling between θ_{+} and θ_{-} might largely shrink the regime of the vestigial phases in the phase diagram.
- The special \tilde{C}_{2n}^1 symmetry in the TB-QC forbids the term $\nabla_{\pm}\theta_{+} \cdot \nabla_{\pm}\theta_{-}$, and favors the formation of the vestigial phases.

Experiment Relevance

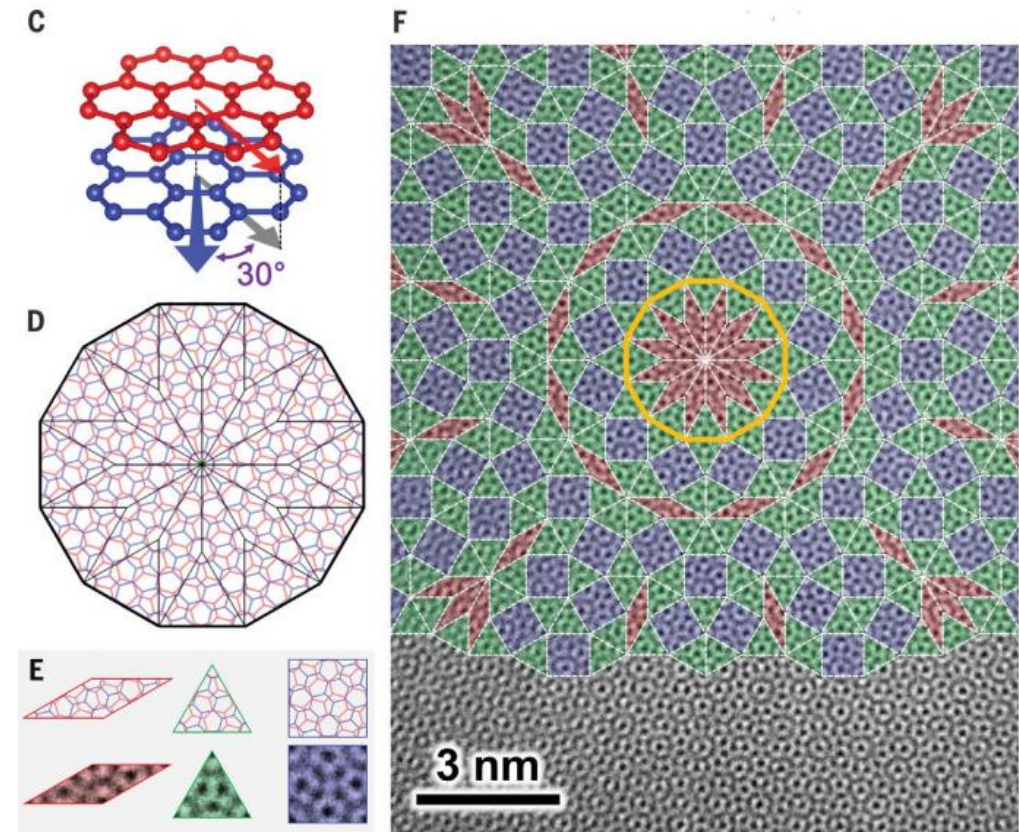
- Synthesis of 45°-twisted bilayer cuprates and 30°-twisted bilayer graphene



Y. Zhu, et al,
Phys. Rev. X 11, 031011 (2021)

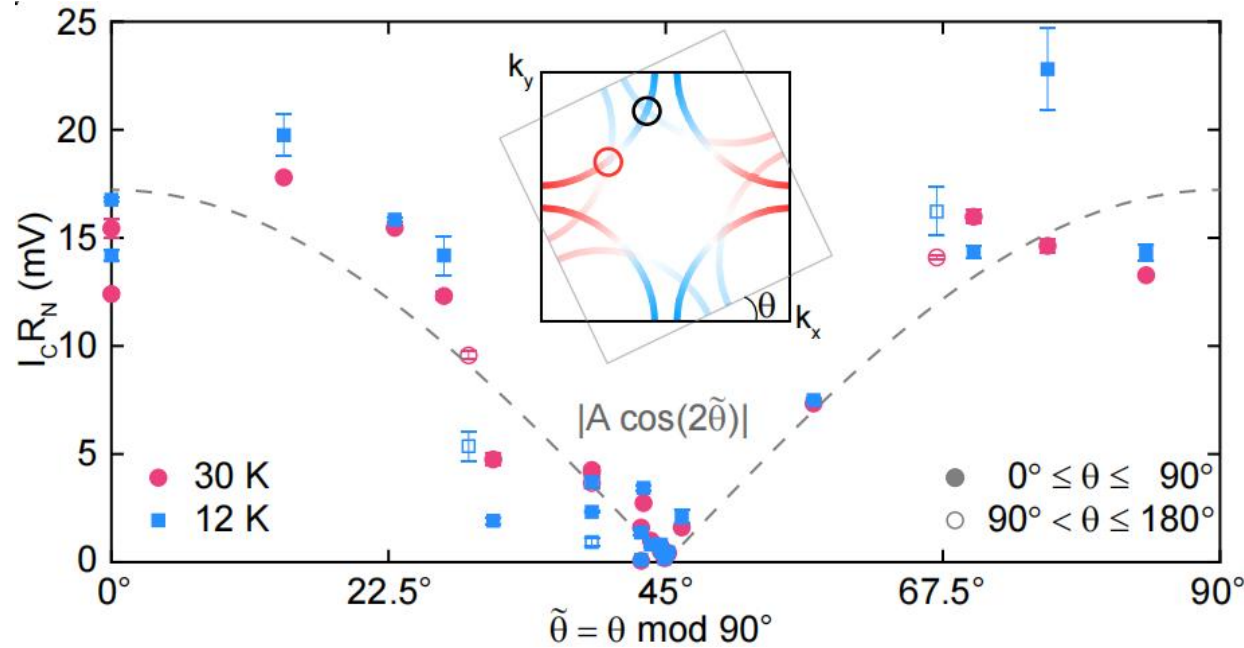


S. Y. Frank Zhao, et al, arXiv: 2108. 13455

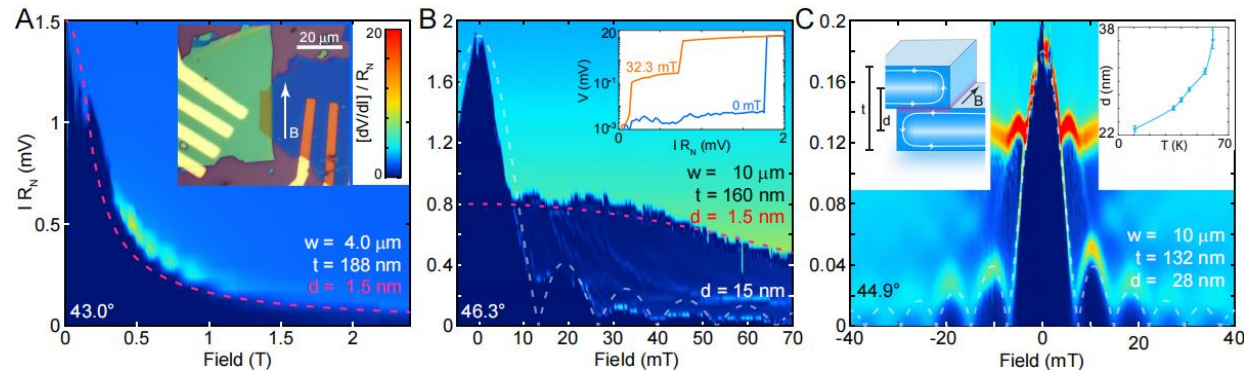


Ahn, et al., Science 361, 782–786 (2018)
W. Yao, et al, PNAS 115, 6928 (2018).

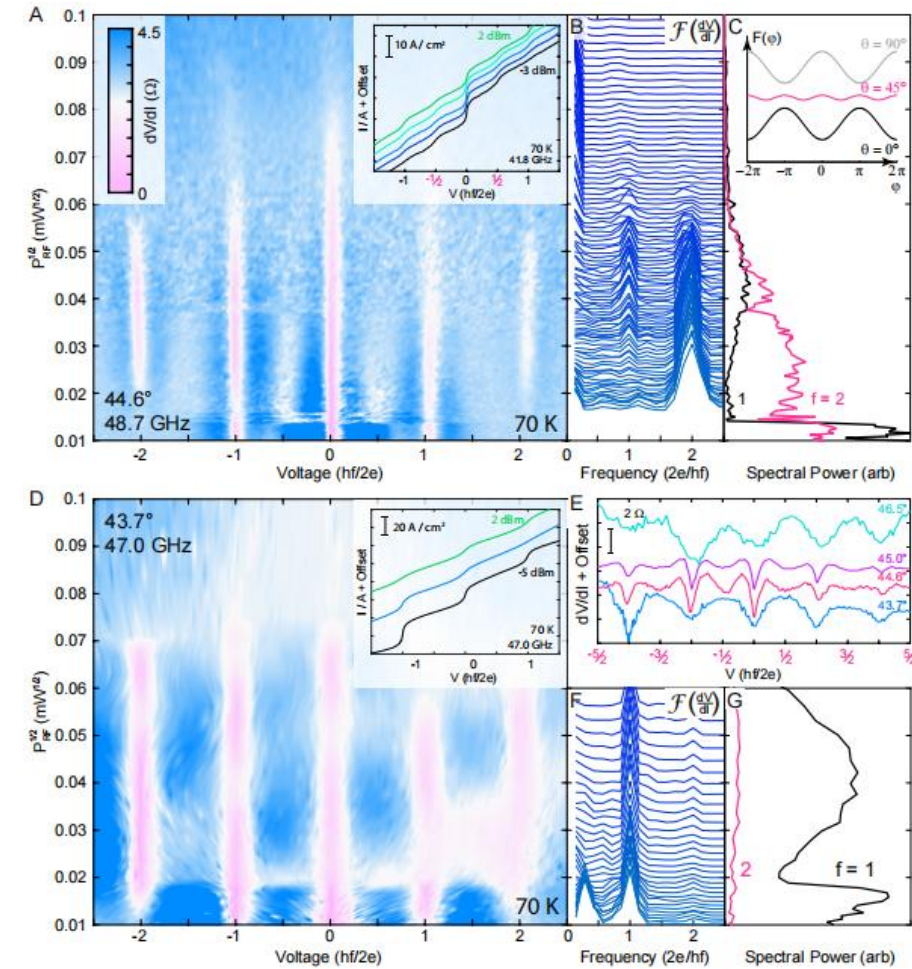
Experiment for the 45°-twisted bilayer cuprates



Twist-angle dependence of the critical current



Fraunhofer pattern near 45°

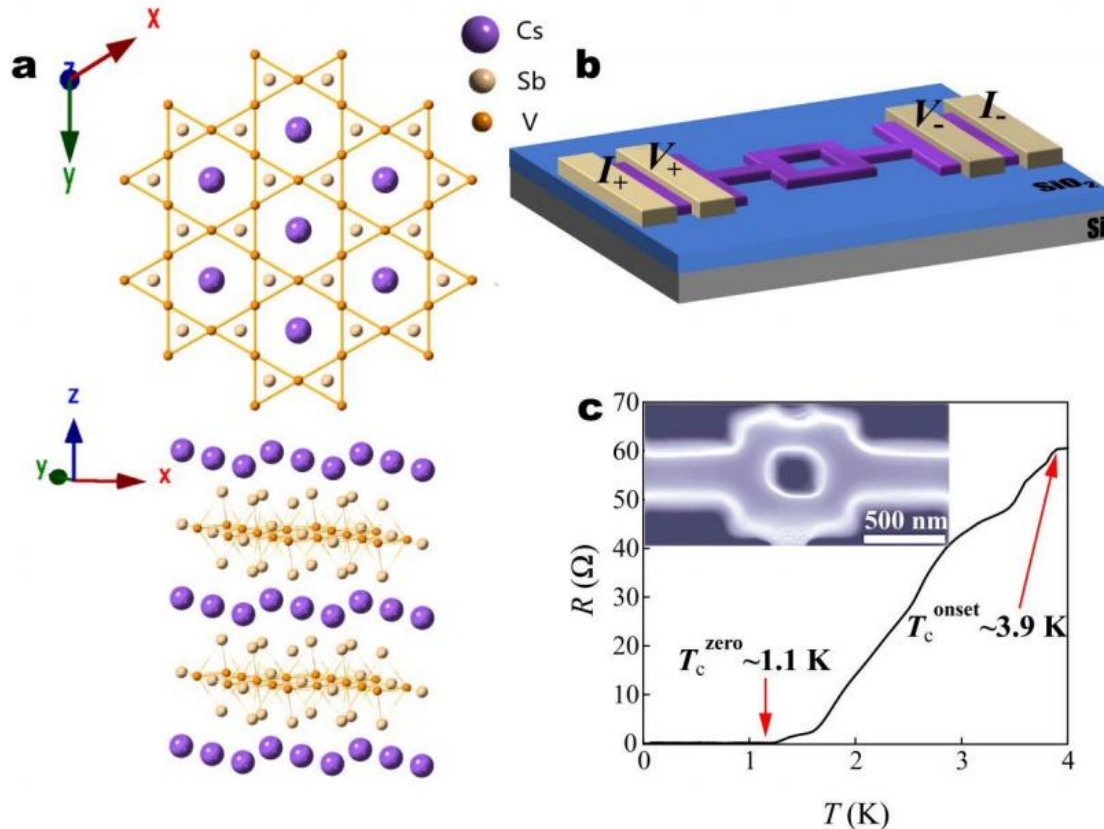


Half-integer Shapiro steps emerge close to 45°

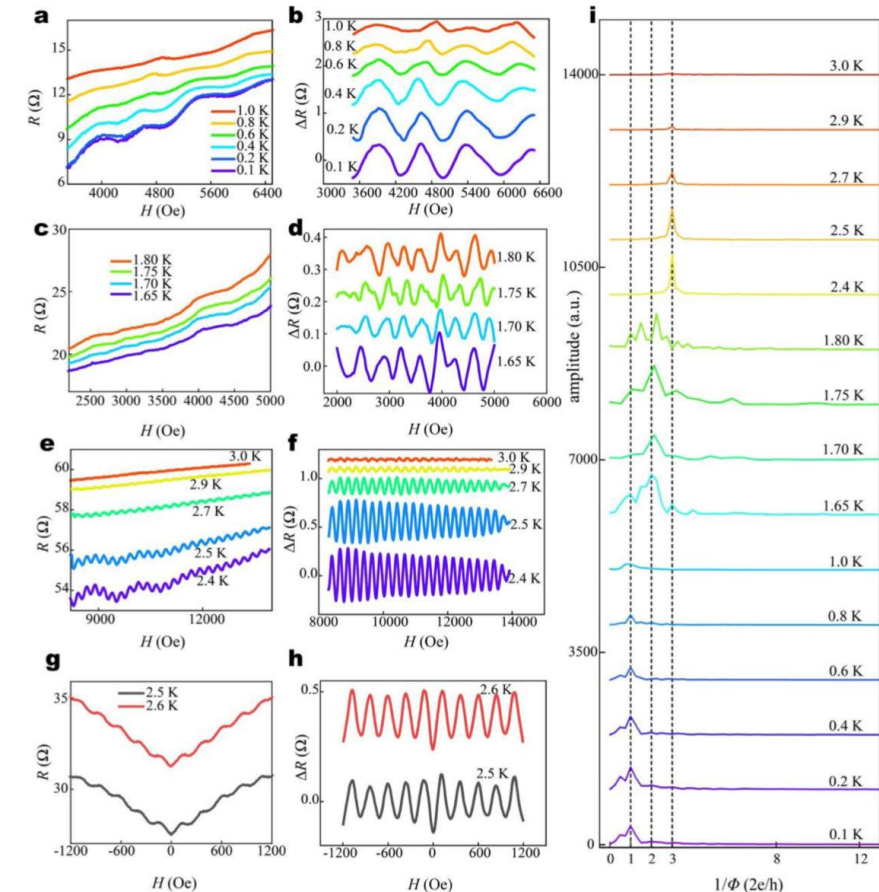
S. Y. Frank Zhao, et al, arXiv: 2108. 13455

How to detect charge-4e SC ?

- The Little-Parks experiment can detect the charge-4e/6e SC



Little-Parks experiment done for CsV_3Sb_5 ,
J. Ge, et al, arXiv:2201.10352



Resistance oscillations in the Little-Parks EXP

Experimental identification of the chiral metal

ARTICLES

<https://doi.org/10.1038/s41567-021-01350-9>

nature
physics



State with spontaneously broken time-reversal symmetry above the superconducting phase transition

Vadim Grinenko ^{1,2,3} ✉, Daniel Weston⁴, Federico Caglieris ², Christoph Wuttke ², Christian Hess ^{2,5},
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Conclusion

- We predict realization of $d+id/f+if$ chiral TSC in the TB-QC
- The charge-4e SC or chiral metal as vestigial phases of the chiral TSC
- The vestigial phases are driven by pairing phase fluctuations
- Locking of the total pairing phase of the two layers: charge-4e SC
- Locking of the relative pairing phase : the chiral metal
- Might apply to other chiral TSCs, but with shrinked vestigial-phase regimes
- We appeal experimental realization of the prediction.

Thanks!